Limitations of the Meta-Reduction Technique: The Case of Schnorr Signatures

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(Informal) Main Results[1]

- Schnorr Signatures are provably secure under the DLOG assumption in the weakly programmable ROM.
- Under the one-more DLOG assumption there does not exist a "single instance" reduction from the DLOG assumption in the non-programmable ROM.
- Eliminating the one-more DLOG assumption from our meta-reduction is highly unlikely.

[1] actual results may vary

Schnorr Signatures [Sch90,Schn91]

$$\mathbb{G} = \langle g \rangle, H$$

$\underline{\mathsf{Kgen}(1^\kappa)}$	$\underline{\mathsf{Sign}(\mathsf{sk}, m)}$	$\underline{\mathsf{Vrfy}(\mathsf{pk}, m, \sigma)}$
$\mathsf{sk} \xleftarrow{\$} \mathbb{Z}_q$	$r \xleftarrow{\$} \mathbb{Z}_q$	parse σ as (c, y)
$\mathsf{pk} := g^{\mathsf{sk}}$	$R := g^r$	if $c \stackrel{?}{=} \mathcal{H}(\mathsf{pk}^{-c} g^y, m)$
return $(\mathsf{sk}, \mathsf{pk})$	$c := \mathcal{H}(R, m)$	output 1
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- Provably secure under DLOG assumption in the ROM [PS96, PS00].
- Previous impossibility results for algebraic reductions [PV05,GBL08,Seu12].

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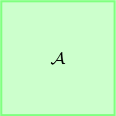
Random Oracle Model
with(out) programmability [FLR+10]



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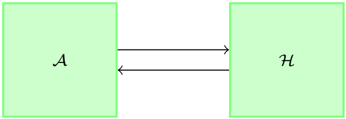

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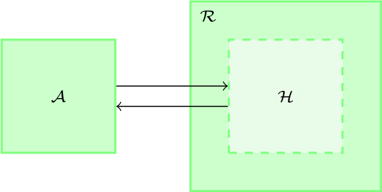

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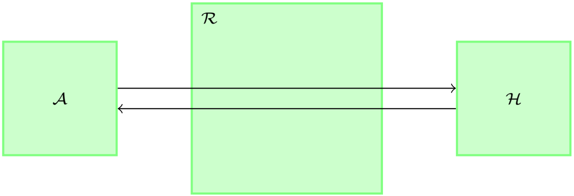

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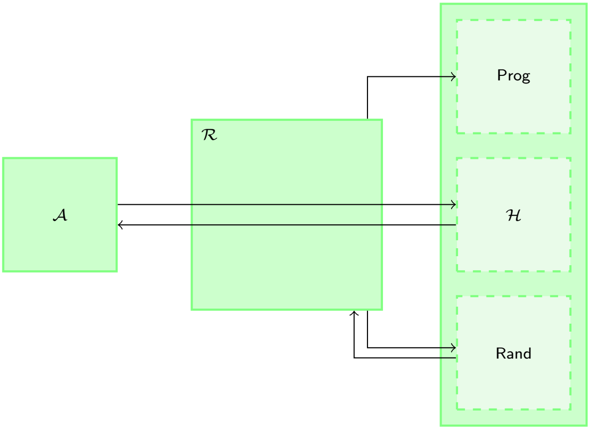

$$\mathsf{Prog}(a, b) \Rightarrow \mathcal{H}(a) \stackrel{\text{def}}{=} \mathsf{Rand}(b)$$

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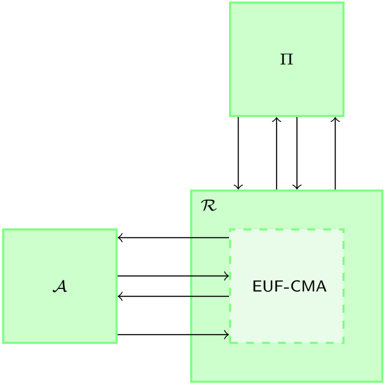
Meta-Reductions [BV98]



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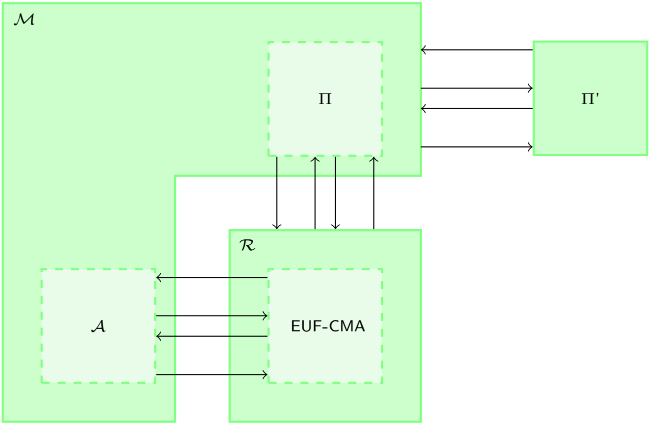

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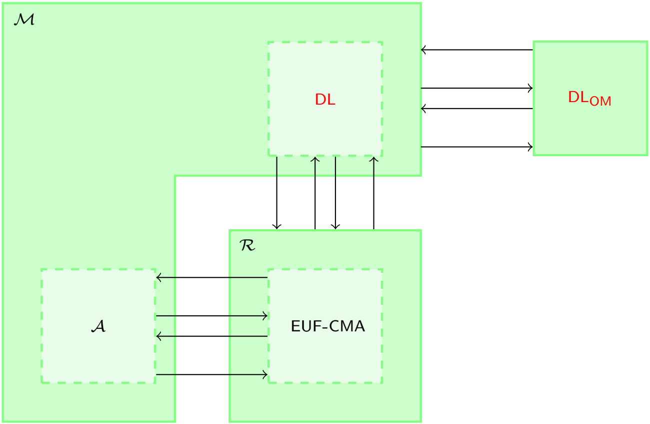

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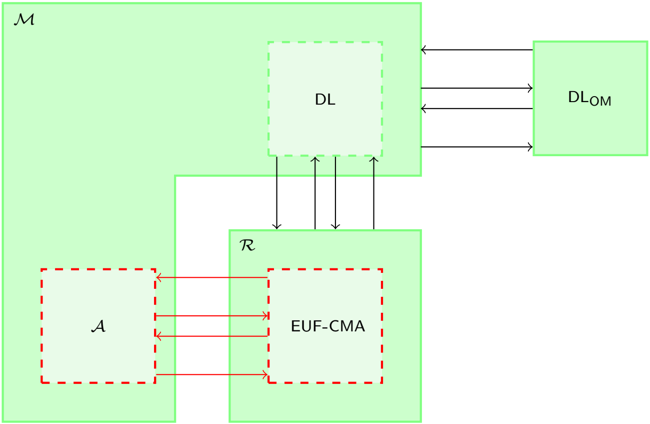


EUF-CMA

pk

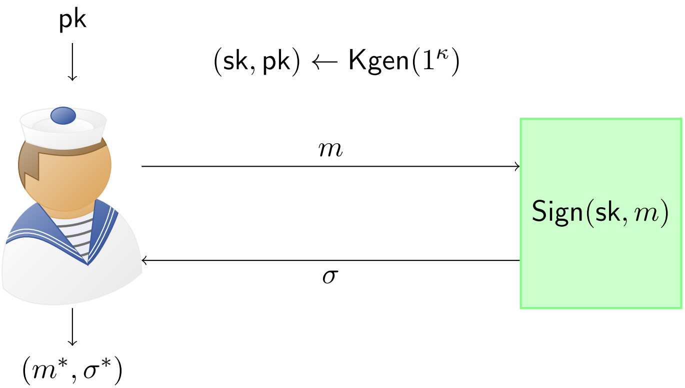
$\downarrow$

$(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{Kgen}(1^\kappa)$

$\downarrow$

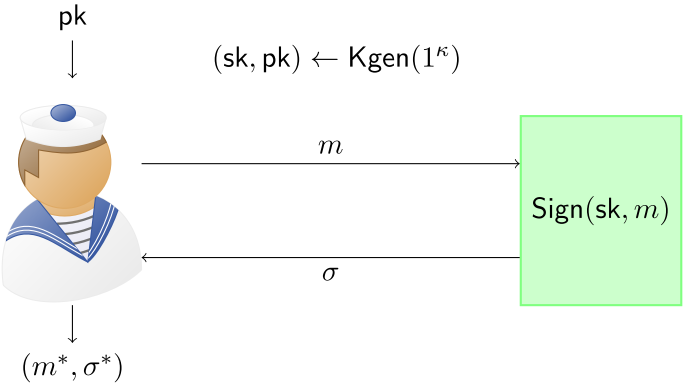
(m^*, σ^*)

EUF-CMA



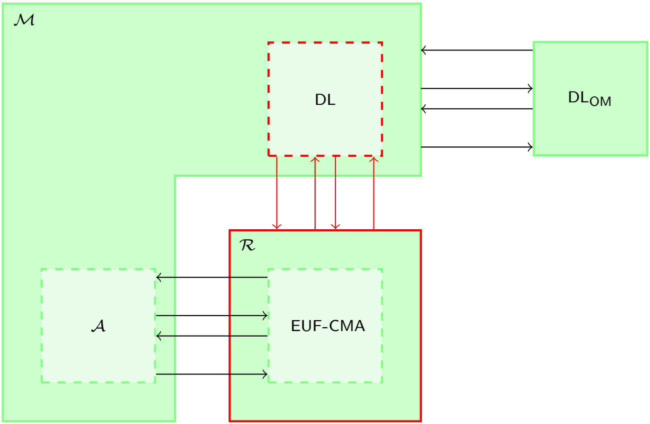
pk
$(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{Kgen}(1^\kappa)$
m
$\mathsf{Sign}(\mathsf{sk}, m)$
σ
(m^*, σ^*)

EUF-CMA



The attacker wins if $\mathsf{Vrfy}(\mathsf{pk}, m^*, \sigma^*) = 1$ and $m \neq m^*$

Meta-Reductions



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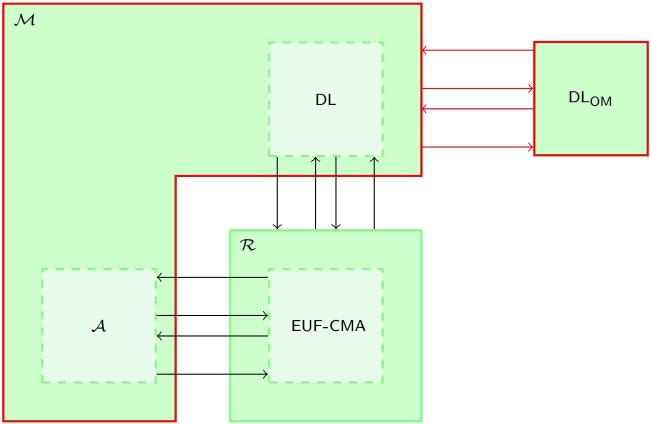

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The One-More discrete log problem [BNPS03]

$$z_1 = g^{x_1}, z_2 = g^{x_2}$$

$$\downarrow$$

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$$x_1, x_2$$

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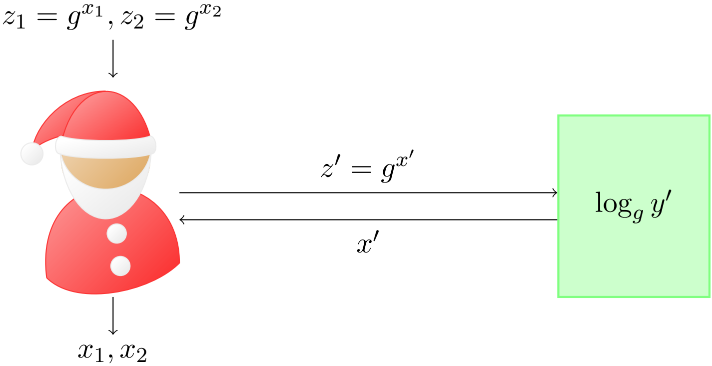

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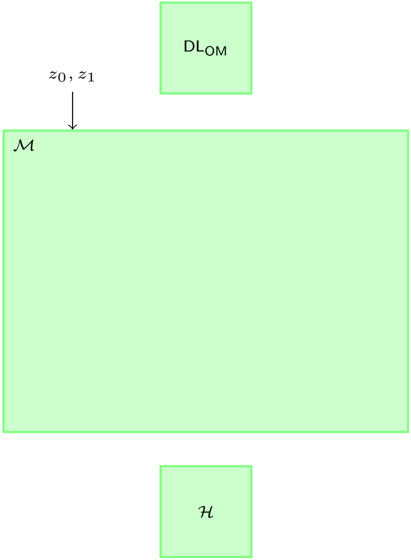

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return $\sigma = (c, y)$

$\underline{\mathsf{Vrfy}(\mathsf{pk}, m, \sigma)}$

parse σ as (c, y)

if $c \stackrel{?}{=} \mathcal{H}(\mathsf{pk}^{-c} g^y, m)$

output 1

else

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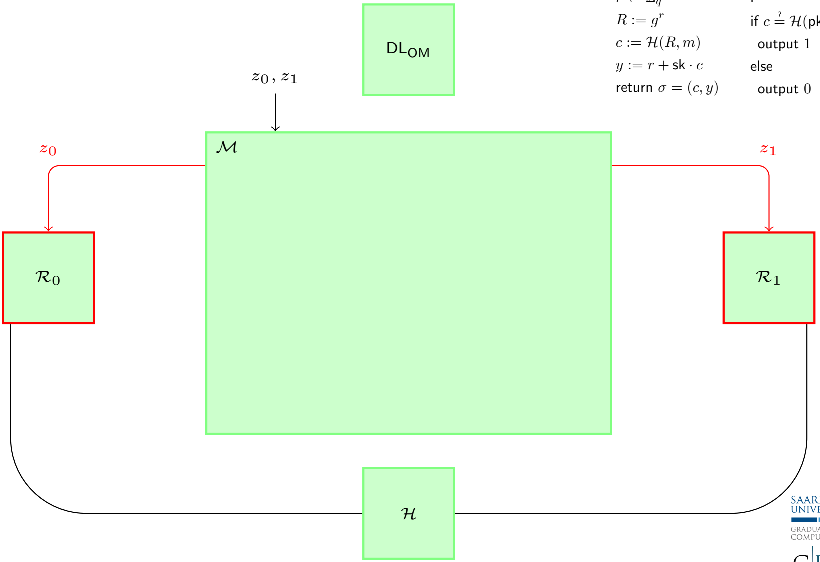

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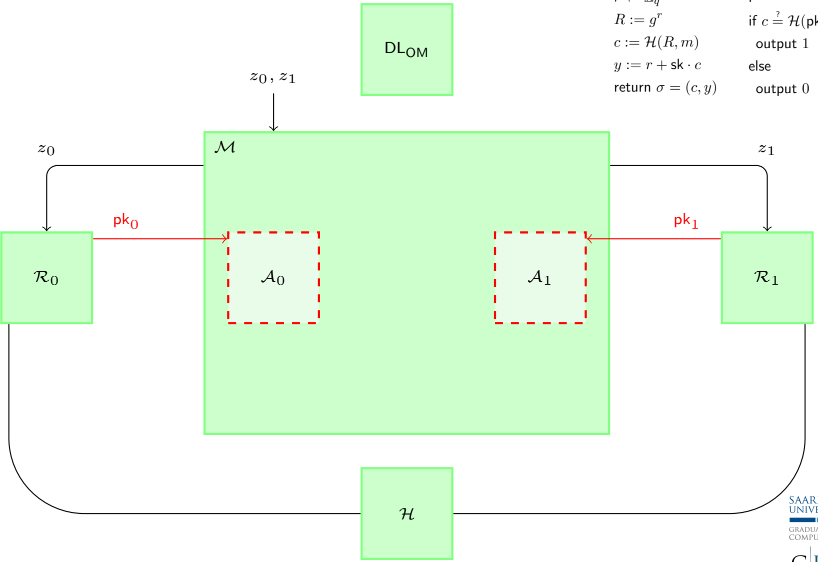

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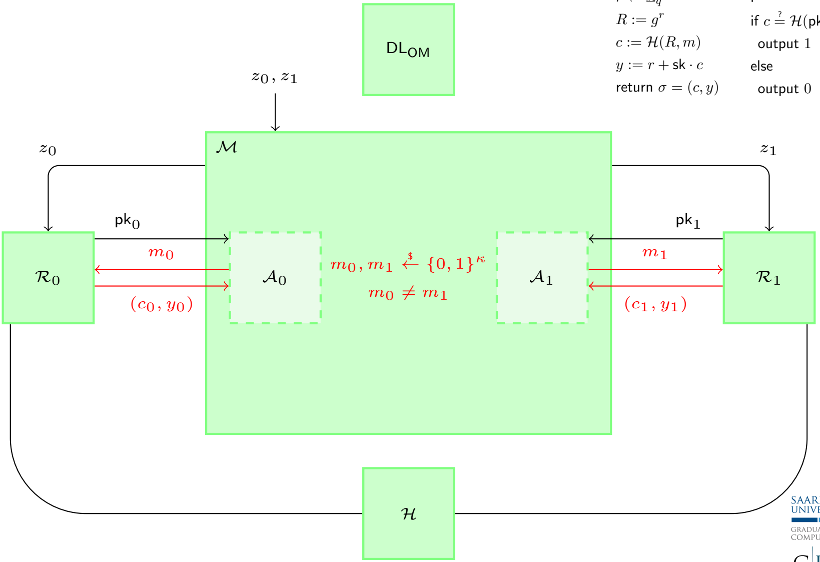

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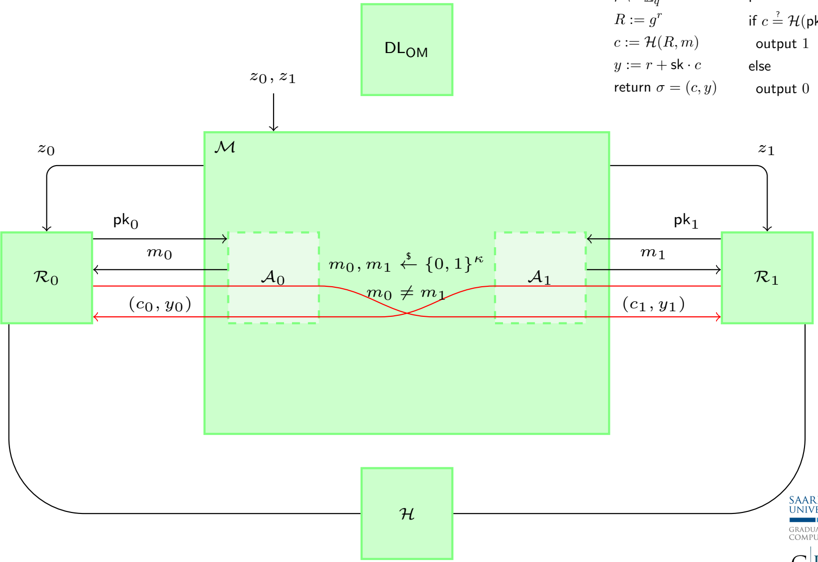

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Sign(sk, m)

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Vrfy(pk, m, σ)

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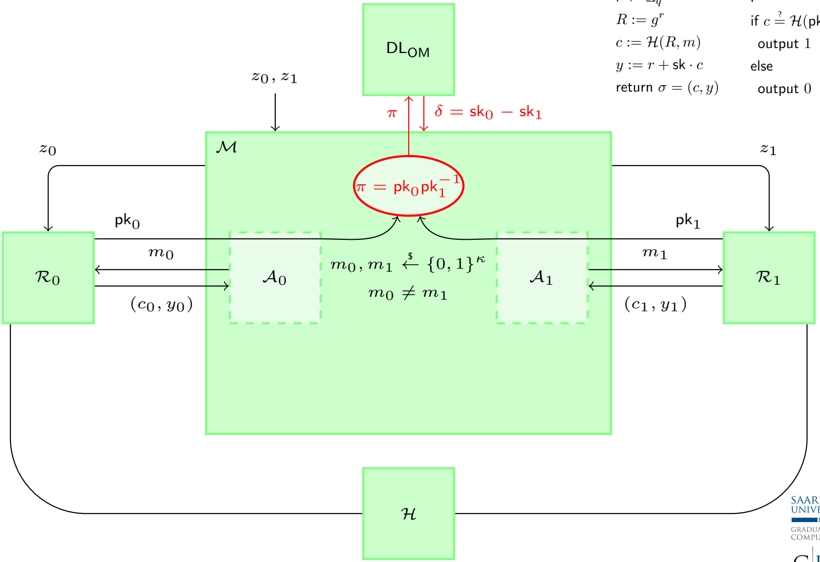

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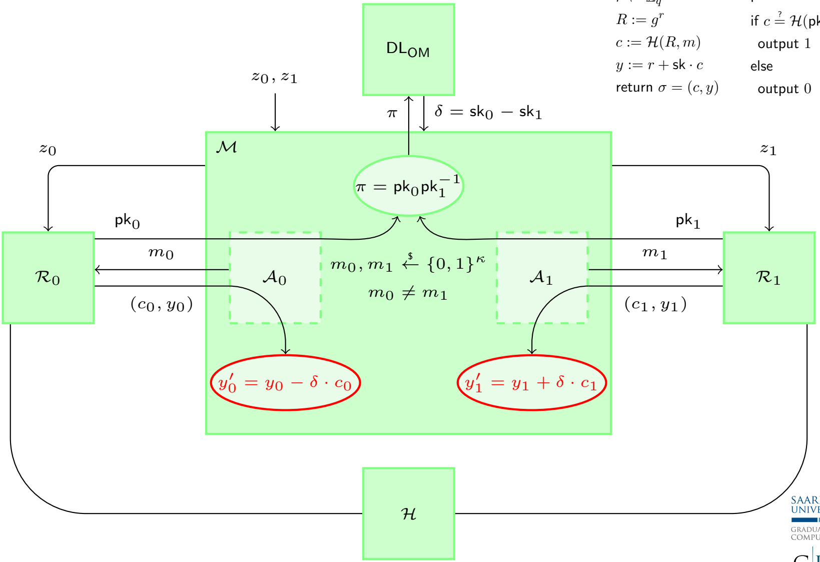

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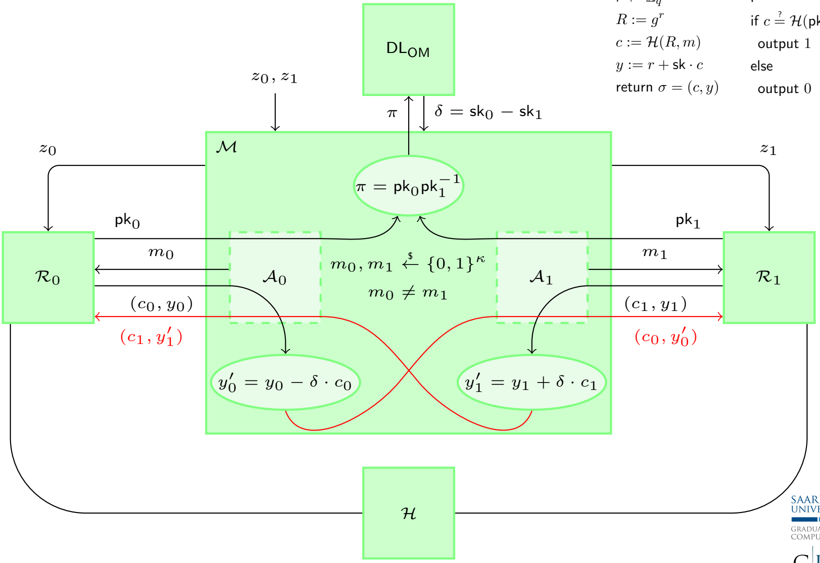

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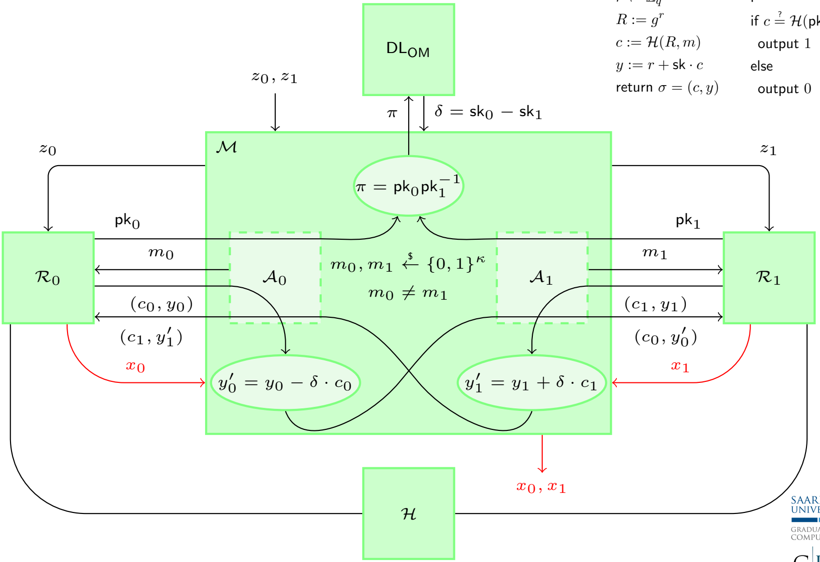

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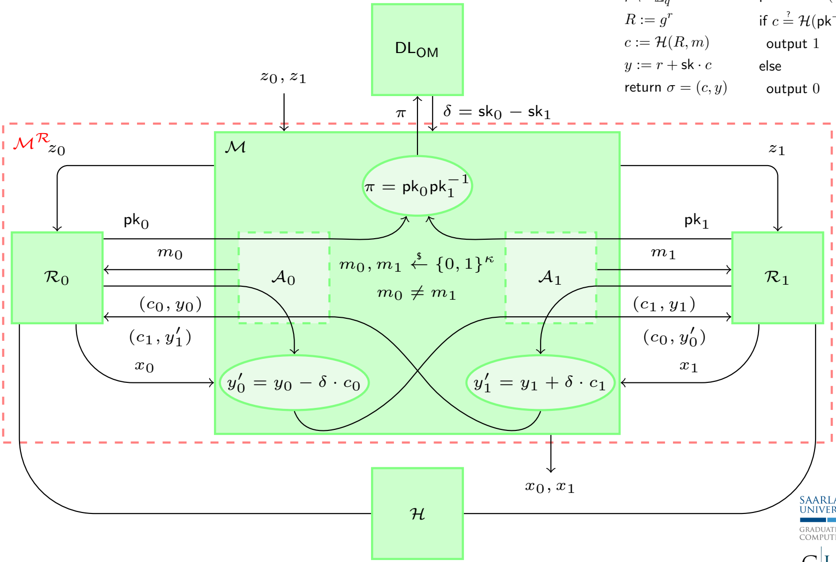

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Can we do better?

- Probably not.
- Going one (meta-)level deeper and using a meta-meta-reduction, we can show that removing the one-more discrete log assumption would (constructively) imply an adversary against the signature scheme.

So, what does this mean?

Under the One-More Discrete Log Assumption, no single instance reductions from the discrete log Problem can exist for Schnorr signatures, if they do not program the random oracle. A relaxed notion of programmability, however, is sufficient.

The result is optimal in the sense that removing the assumption proves to be extremely unlikely.

Open Problems

- We rule out DLOG reductions, but what about CDH,...
- Possibly even interactive assumptions?

Thank You!

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