On Statistically Secure Obfuscation with Approximate Correctness

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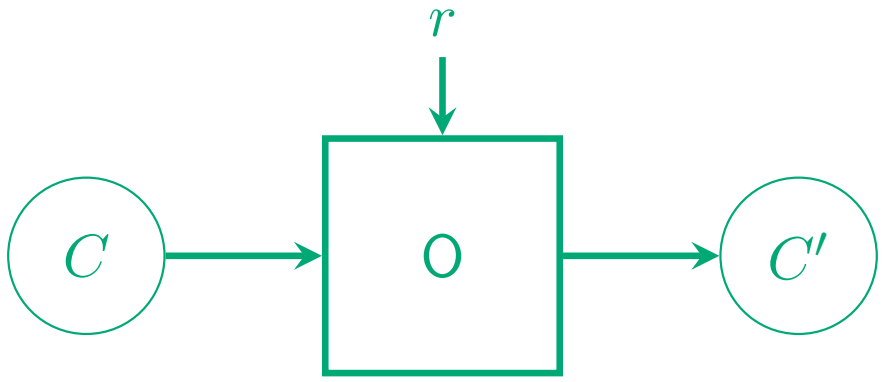
Statistically Secure Obfuscation



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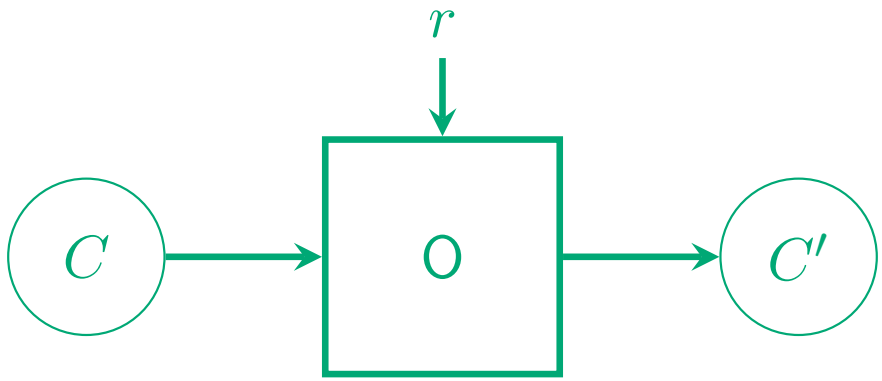
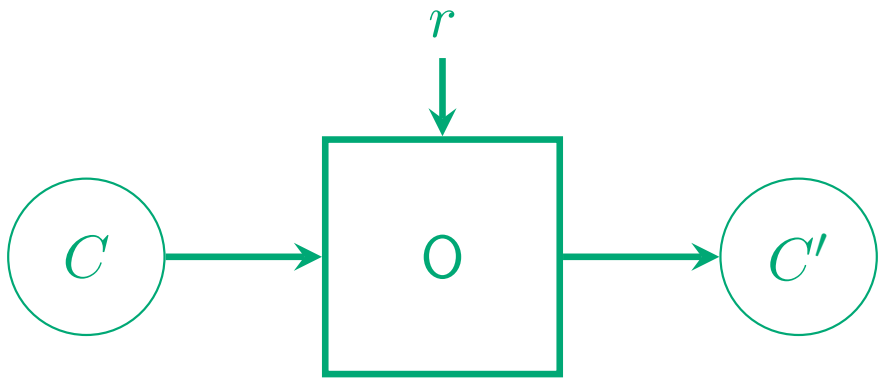


- **Perfect Correctness:** For any circuit C

$$\forall x : C'(x) = C(x)$$

Statistically Secure Obfuscation



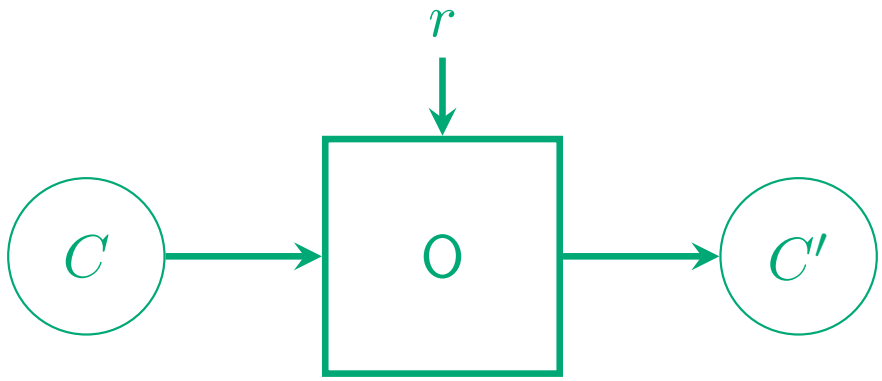
- **Perfect Correctness:** For any circuit C

$$\forall x : C'(x) = C(x)$$

- $(1-\epsilon)$-**Approximate Correctness:** For any circuit C,

$$\Pr_{r,x}\left[C'(x) = C(x)\right] \geq 1 - \epsilon(n)$$

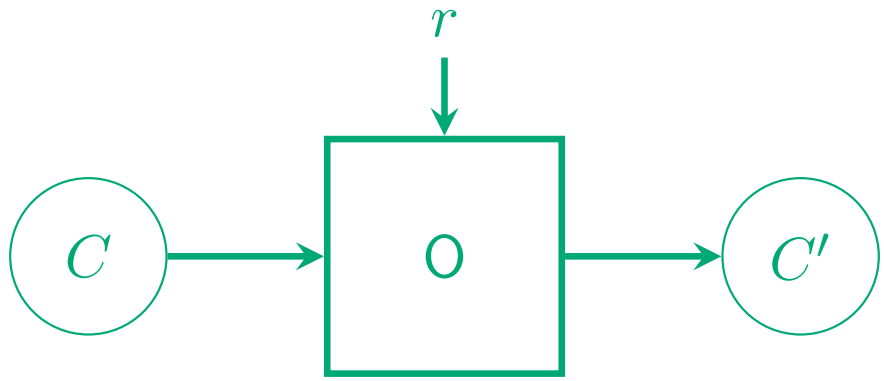
Statistically Secure Obfuscation



- **Indistinguishability Obfuscator:** For any pair of circuits, such that $C_1 \equiv C_2$ and $|C_1| = |C_2|$

$$\mathsf{SD}(\mathsf{O}(C_1), \mathsf{O}(C_2)) \leq \mathsf{negl}(n)$$

Statistically Secure Obfuscation



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- $(1-\delta)$-**Correlation Obfuscator:** For any pair of circuits, such that $C_1 \equiv C_2$ and $|C_1| = |C_2|$

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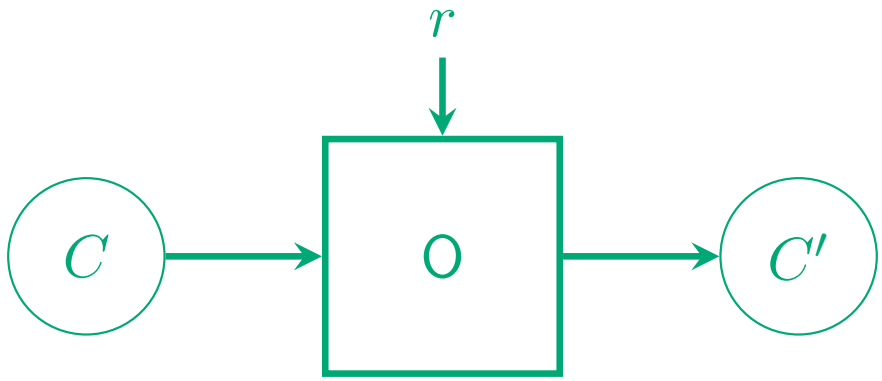
Statistically Secure Obfuscation



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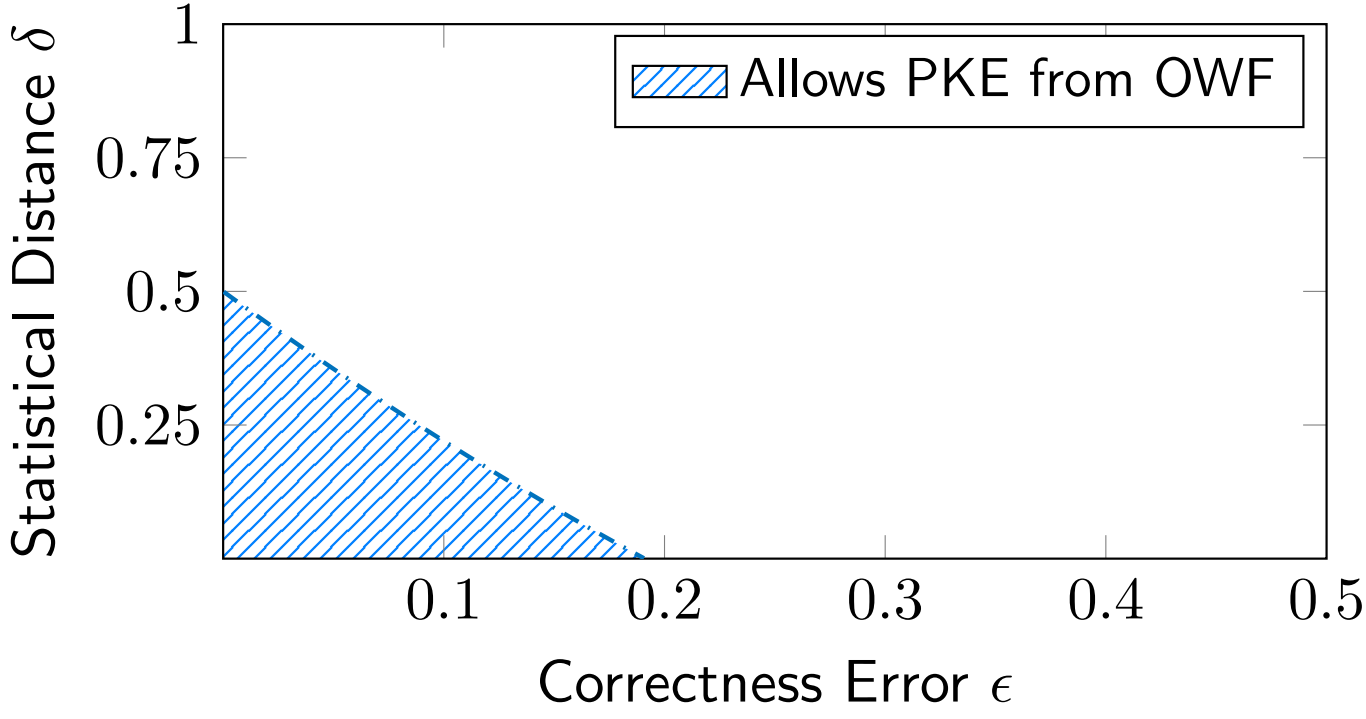
Why Do We Even Care About Approximate Correctness?

Because approximate obfuscation is useful!
[MMNPs16,SW14,Hol06]



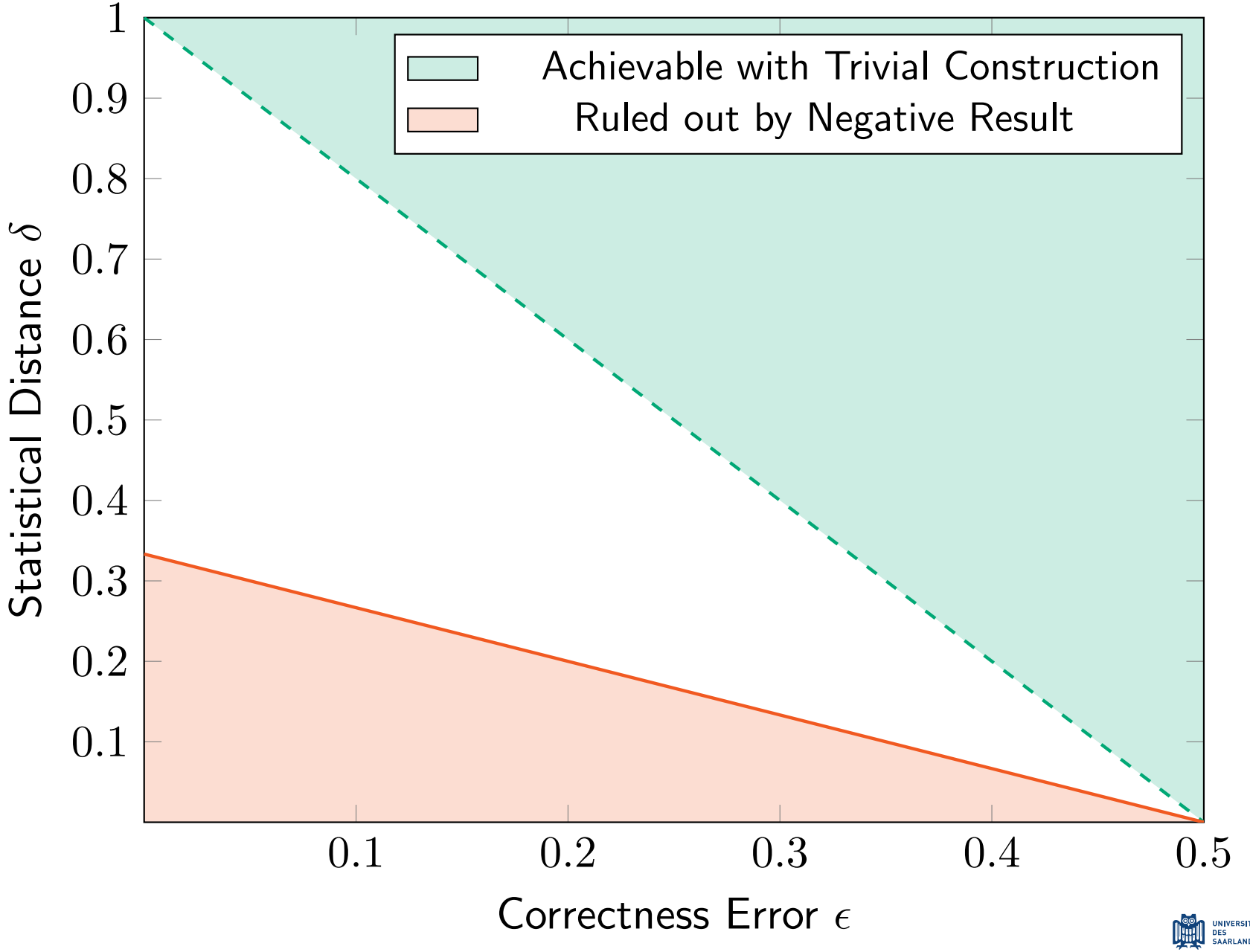
Main Result

- If statistically secure, approximately correct iO (saiO) exists, then either one-way functions do not exist, or NP $\subseteq$ AM $\cap$ coAM.

- **More Generally:** If $(1-\delta)$-statistically secure, $(1-\epsilon)$-approximately correct correlation obfuscation (sacO) exists with $\delta(n) \leq \frac{1}{3} - \frac{2}{3}\epsilon(n) - \frac{1}{\mathsf{poly}(n)}$, then either one-way functions do not exist, or NP $\subseteq$ AM $\cap$ coAM.

- For very weak parameters, a trivial construction of sacO exists with $\delta(n) = 2\epsilon(n)$.

The Landscape of Correlation Obfuscation



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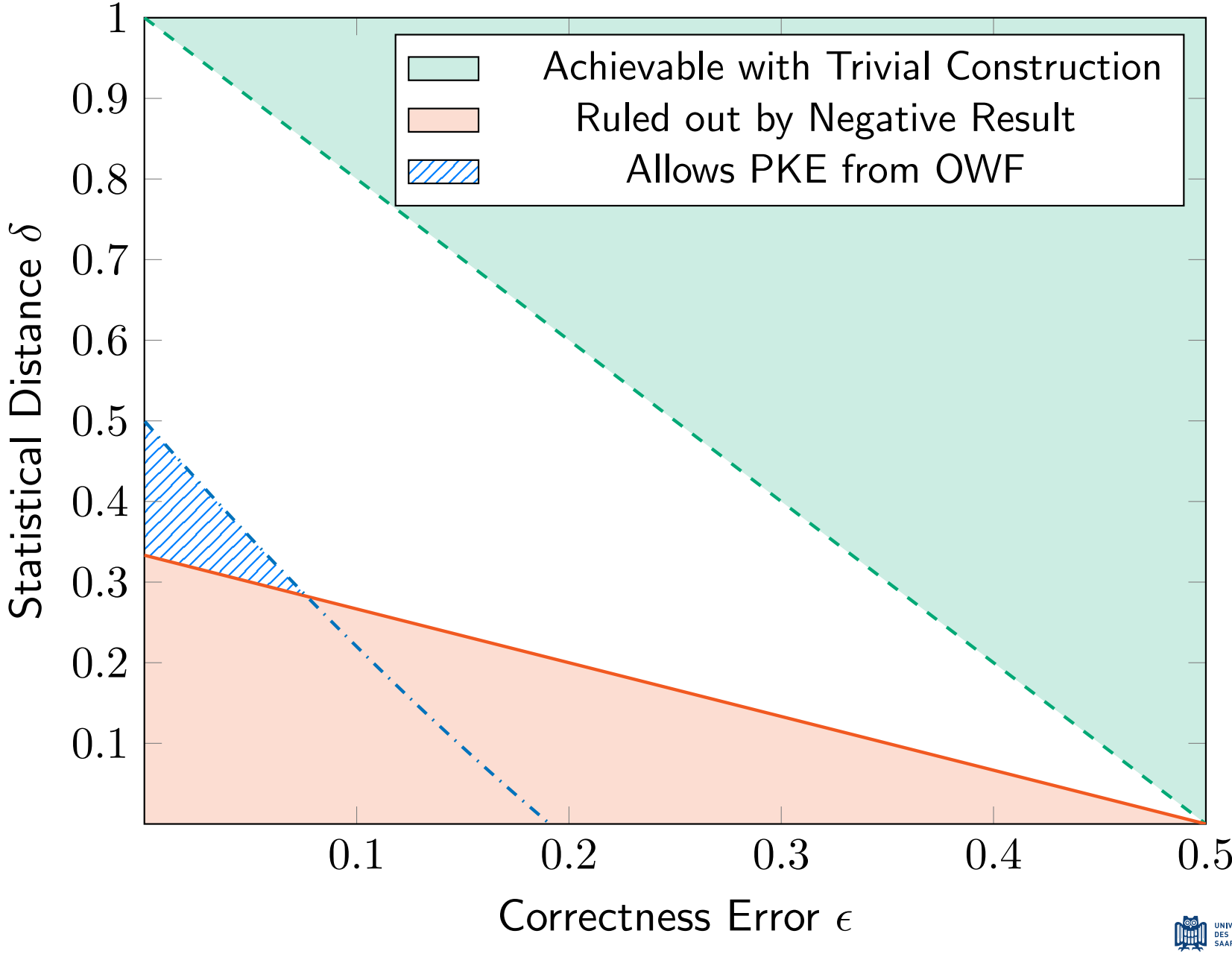

Impossibility of Perfect Correctness [GR07]

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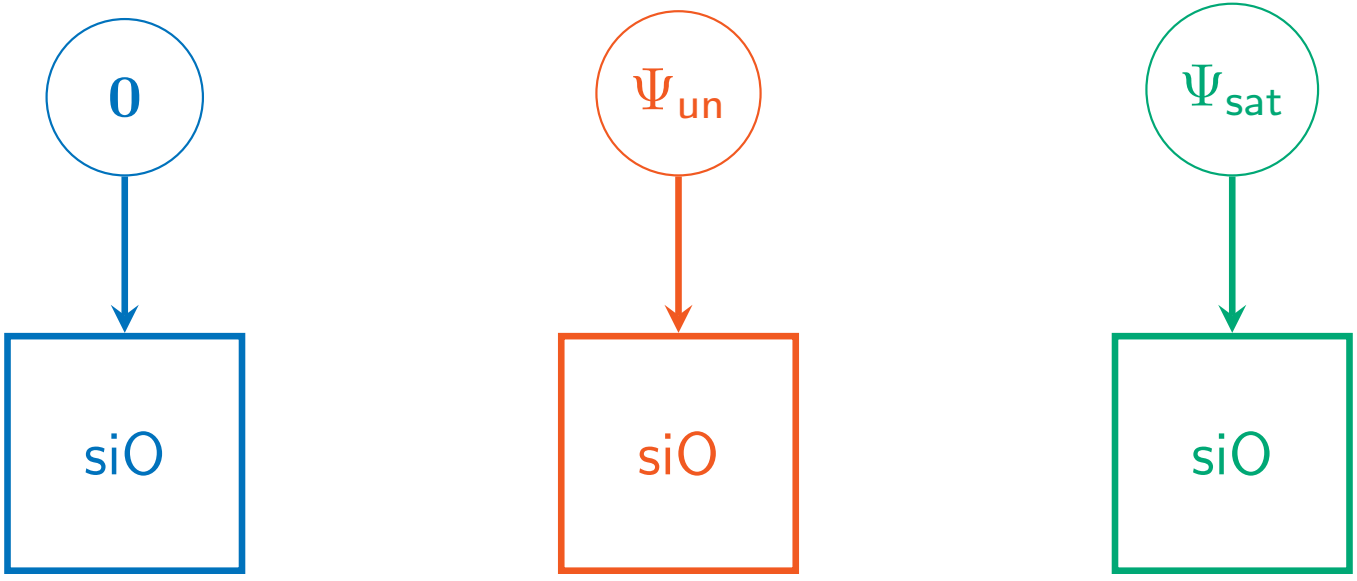

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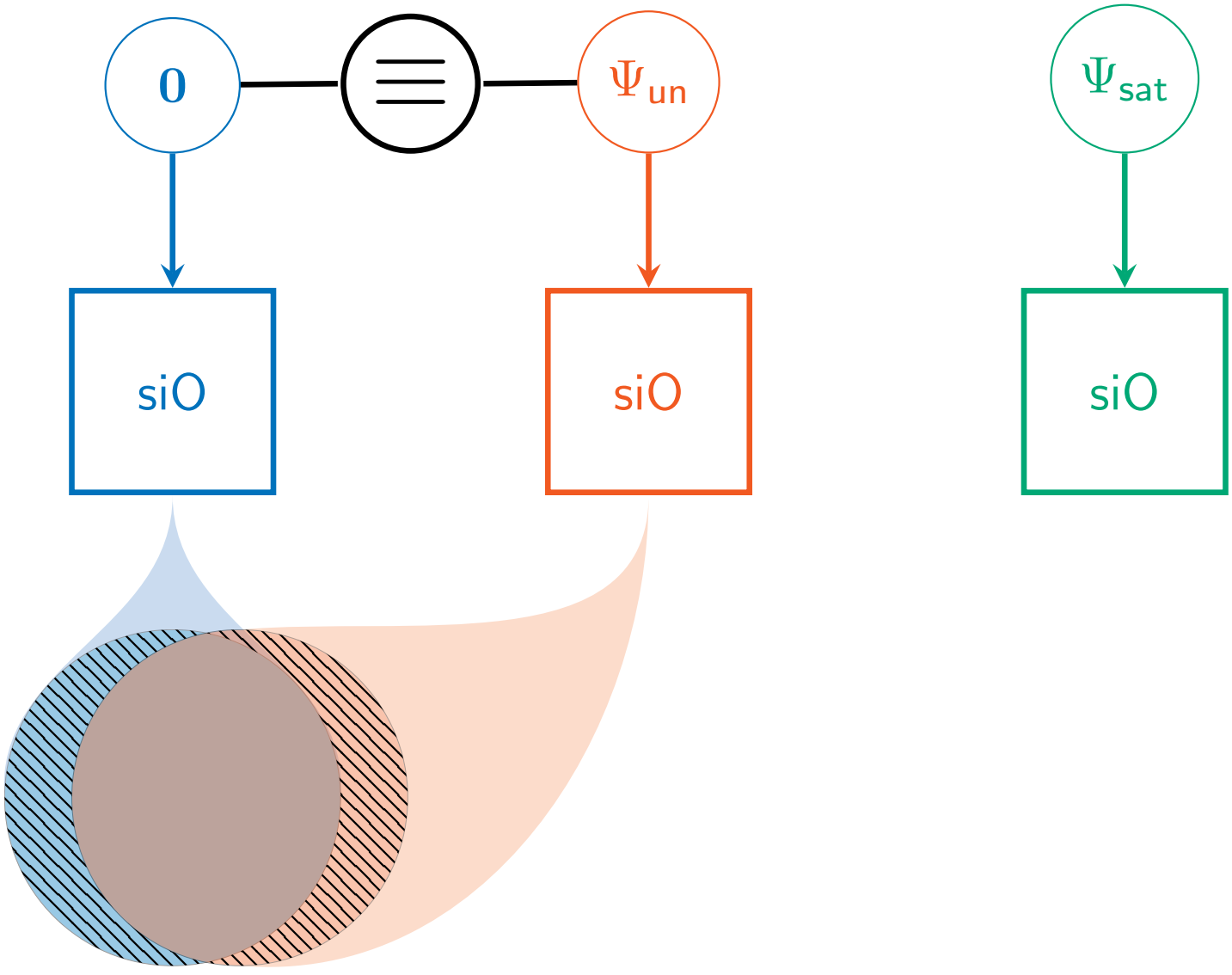

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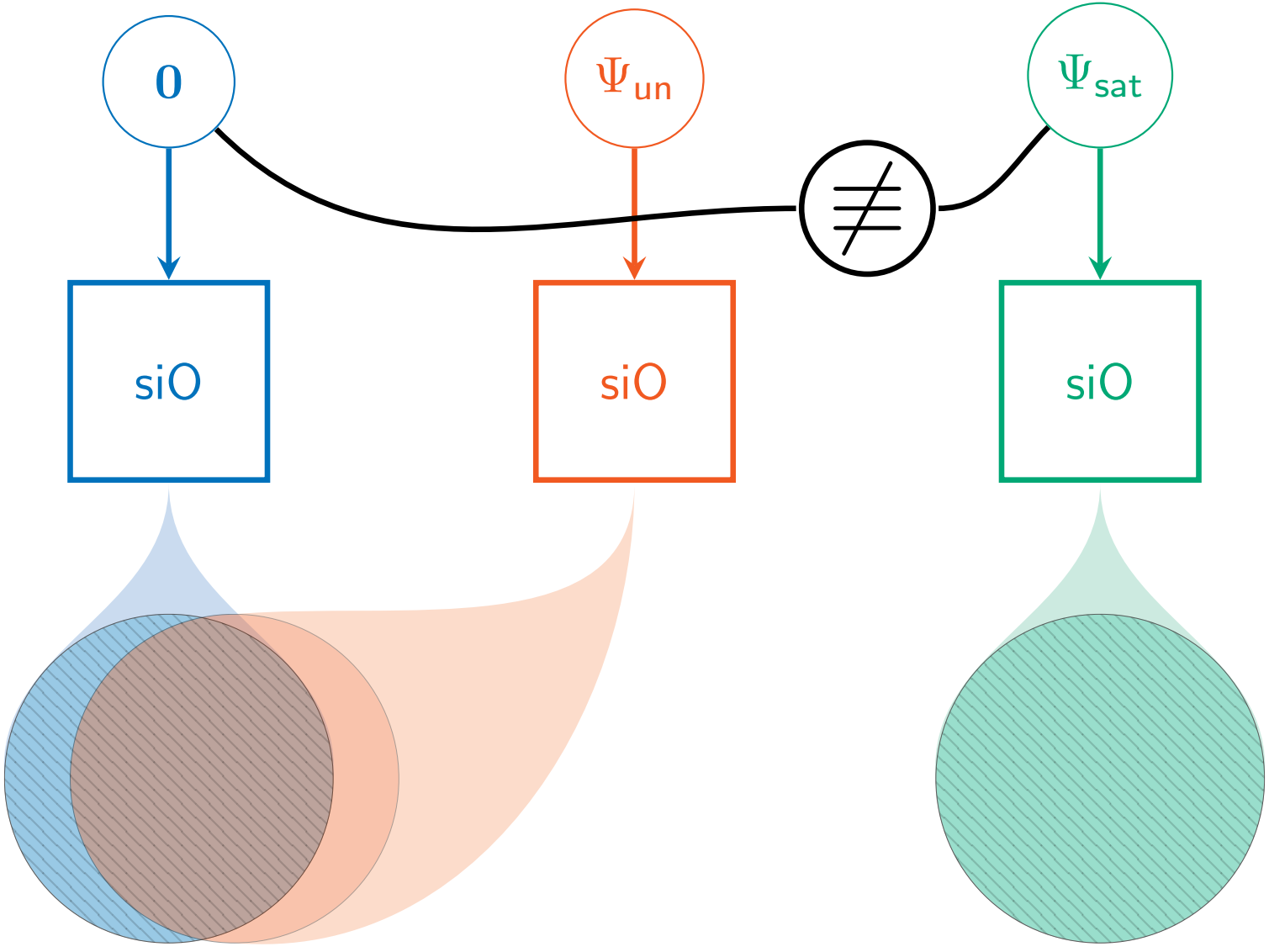

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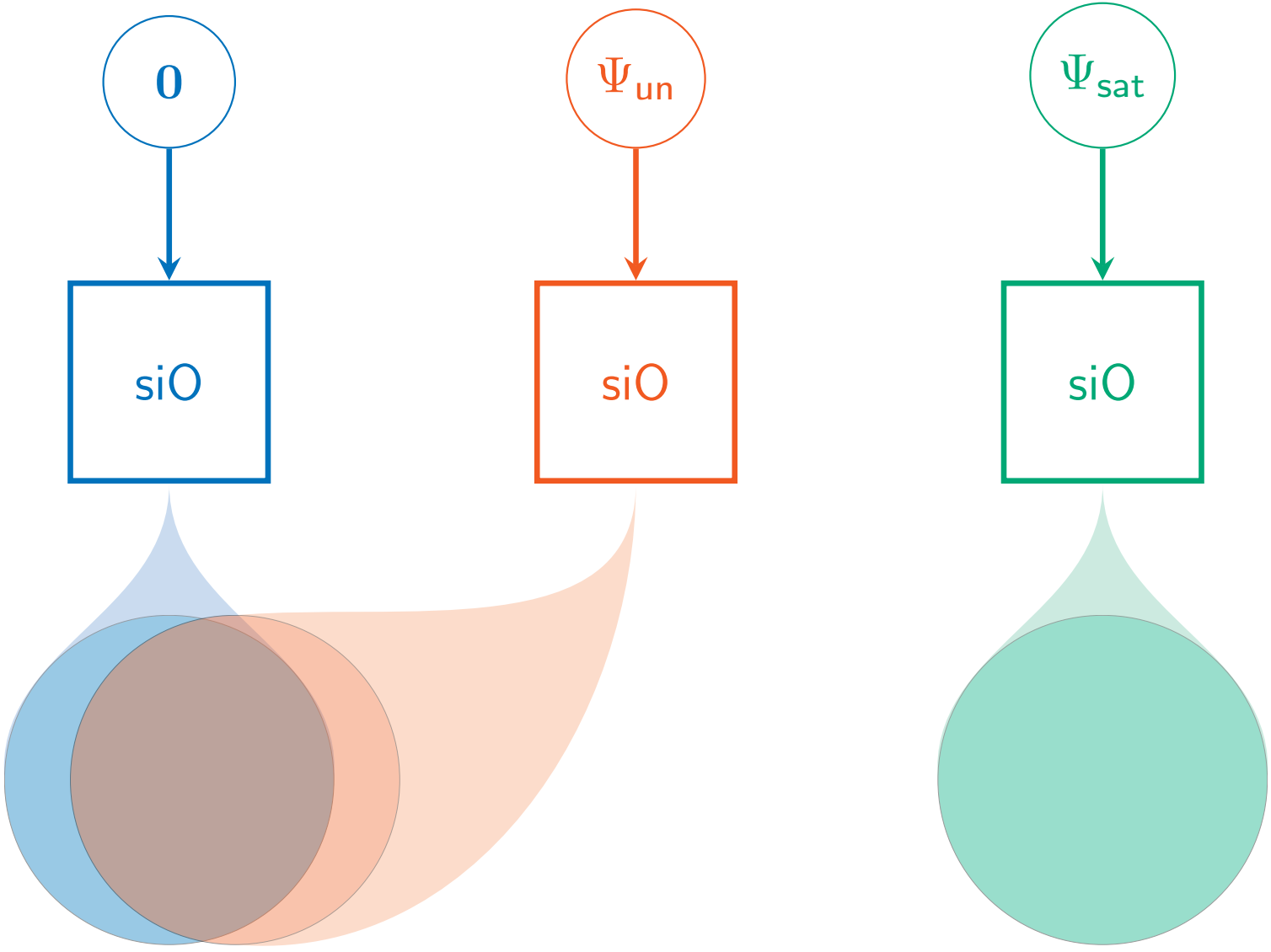
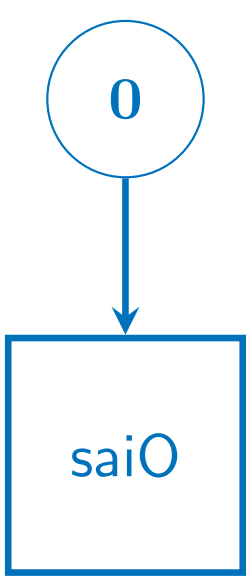
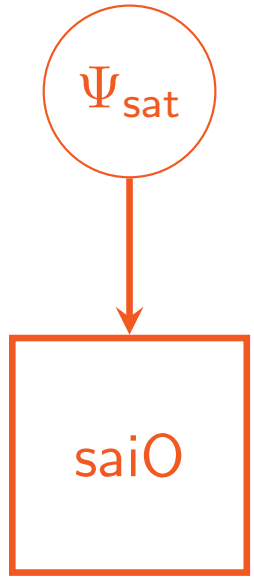


$$\text{GapSD} \in \text{AM} \cap \text{coAM} \implies \text{NP} \subseteq \text{AM} \cap \text{coAM}$$

Why Does the Approach Fail in the Approximate Case?





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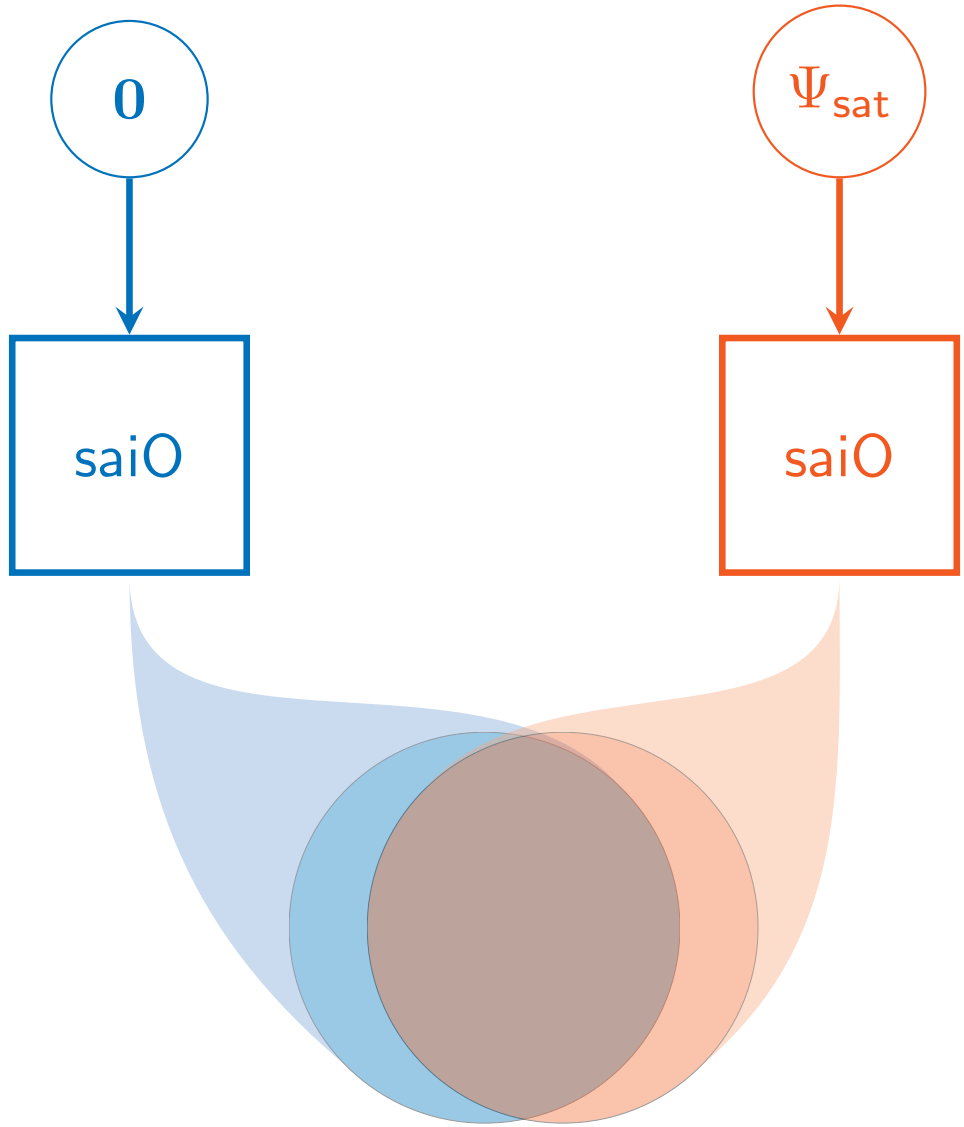
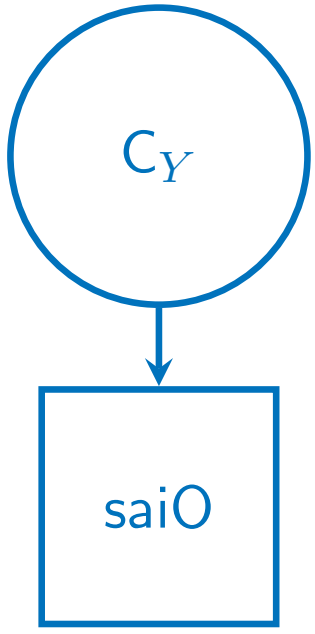
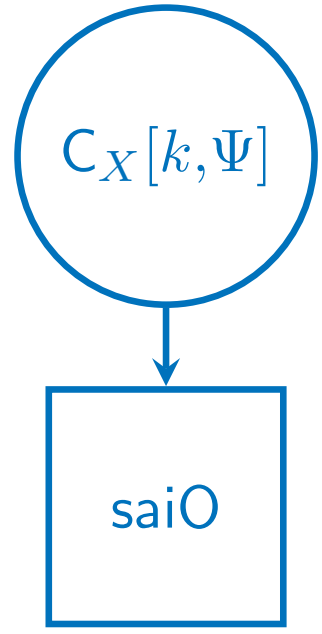


Overview of Our Approach





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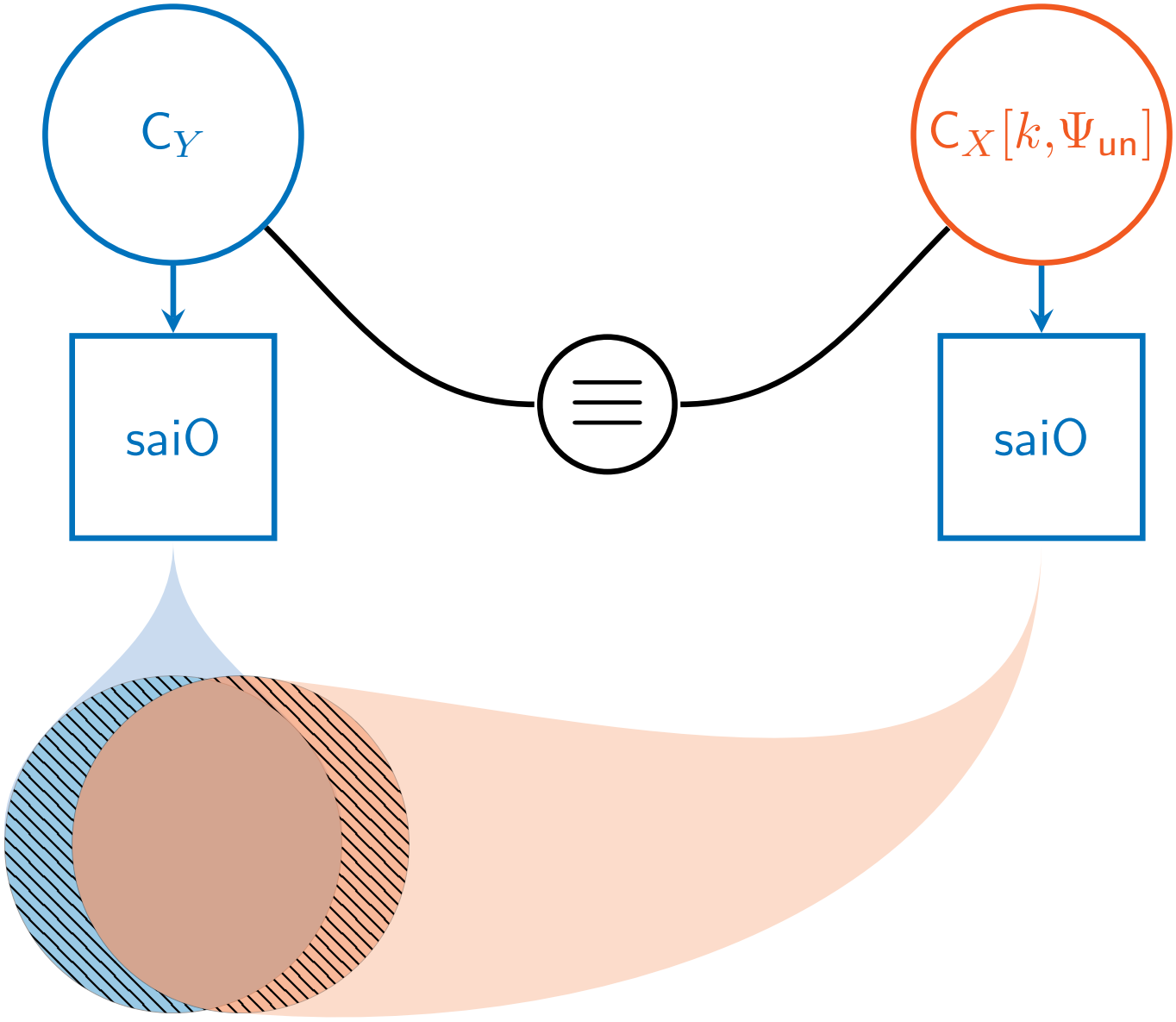

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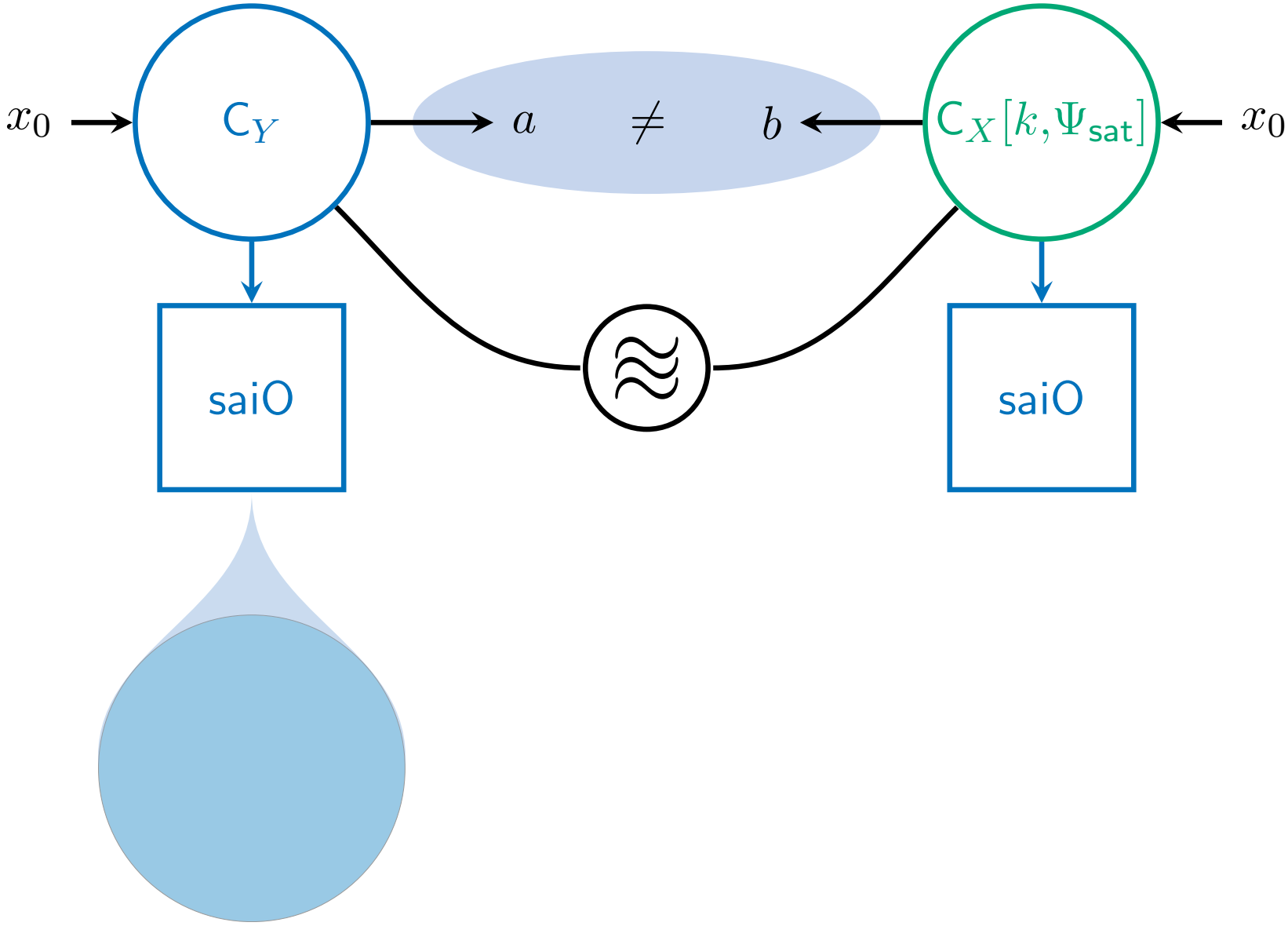

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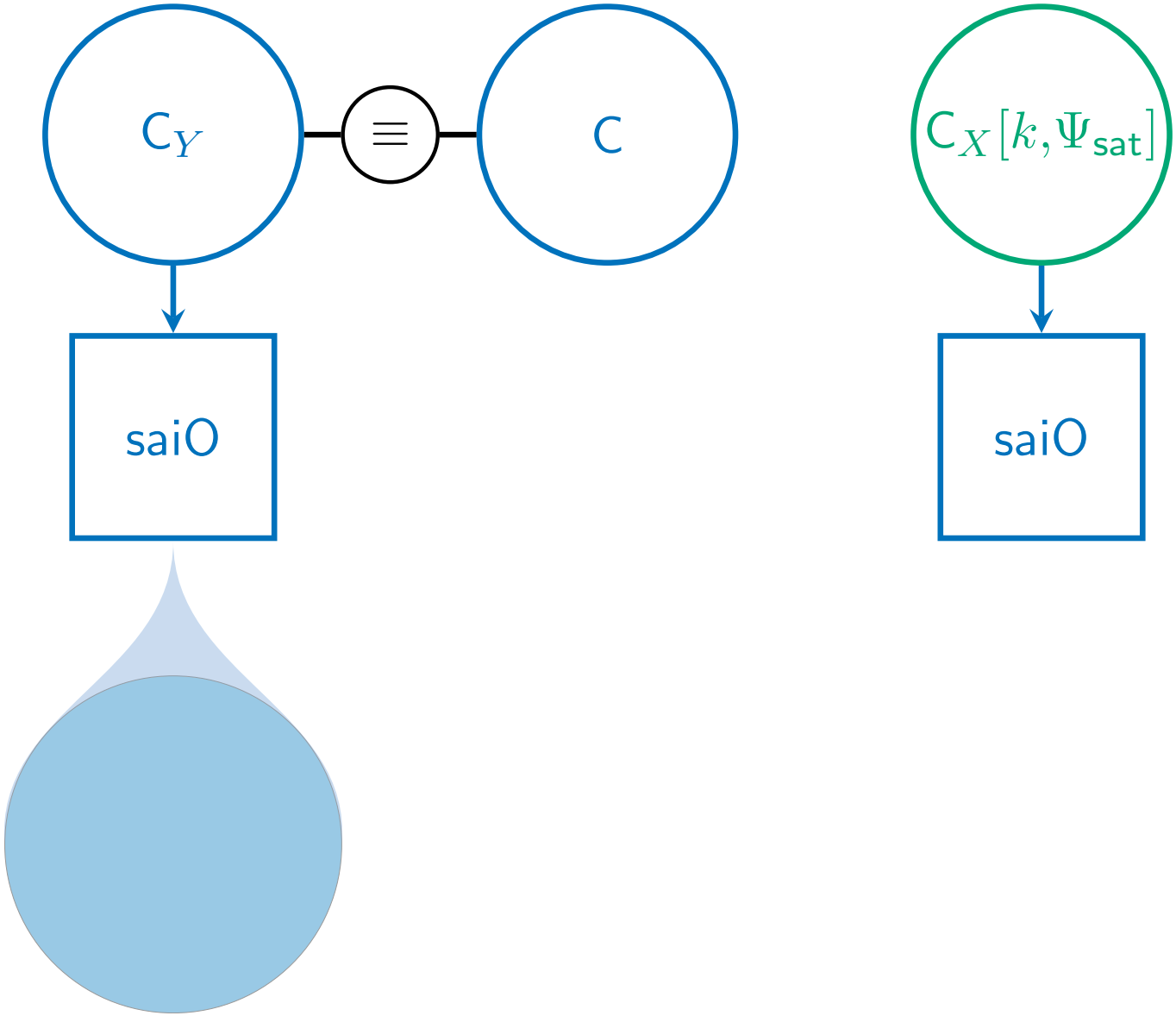

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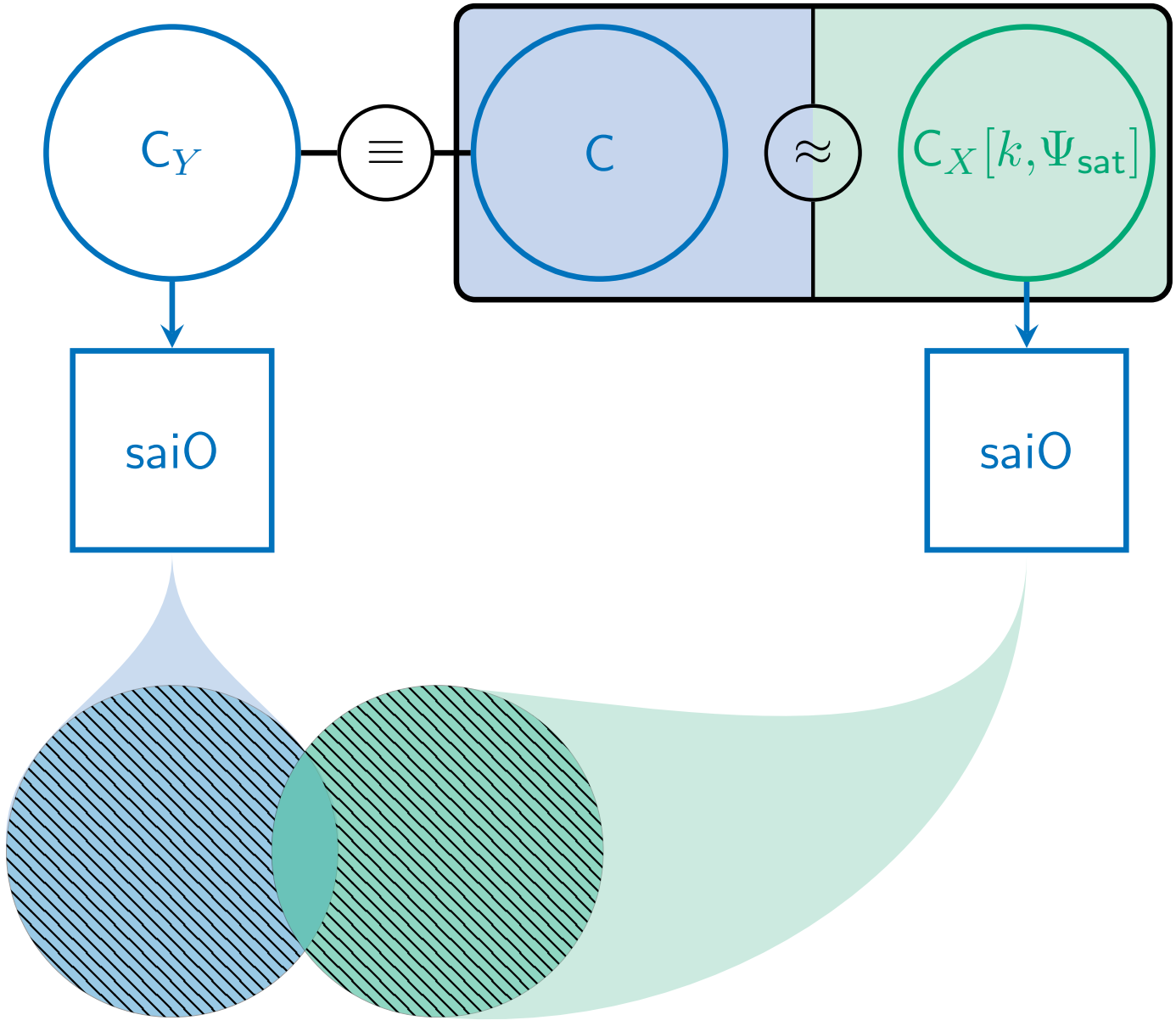

Puncturable Pseudorandom Functions [BW13,BGI14,KPTZ13]

$b \leftarrow \mathsf{PRF}(k, x) \qquad k^* \leftarrow \mathsf{Puncture}(k, x_0)$

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  For all $x \neq x_0$, $\mathsf{PRF}(k^*, x) = \mathsf{PRF}(k, x)$

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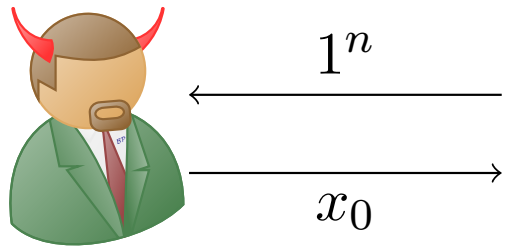

$$k \leftarrow_{\$} \{0,1\}^n, k^* = \mathsf{Puncture}(k, x_0)$$

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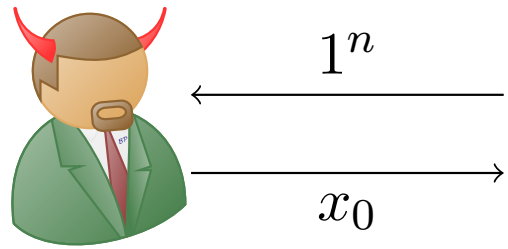

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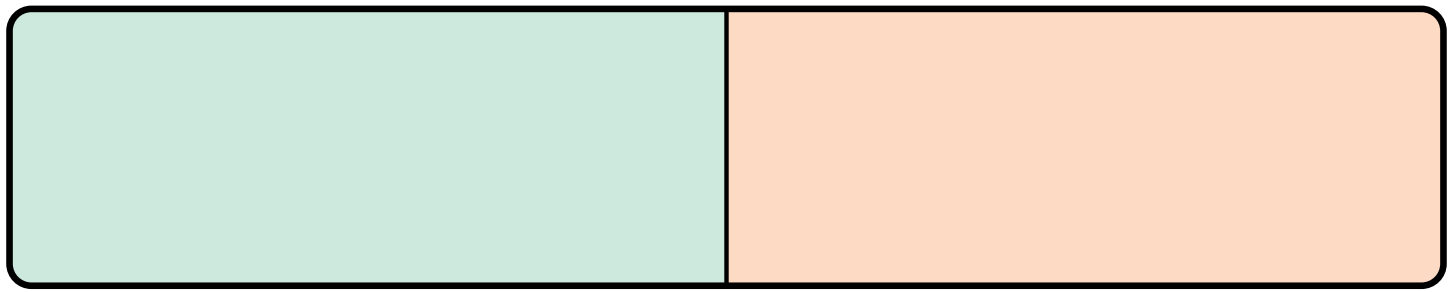

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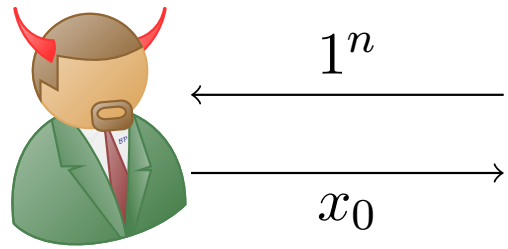

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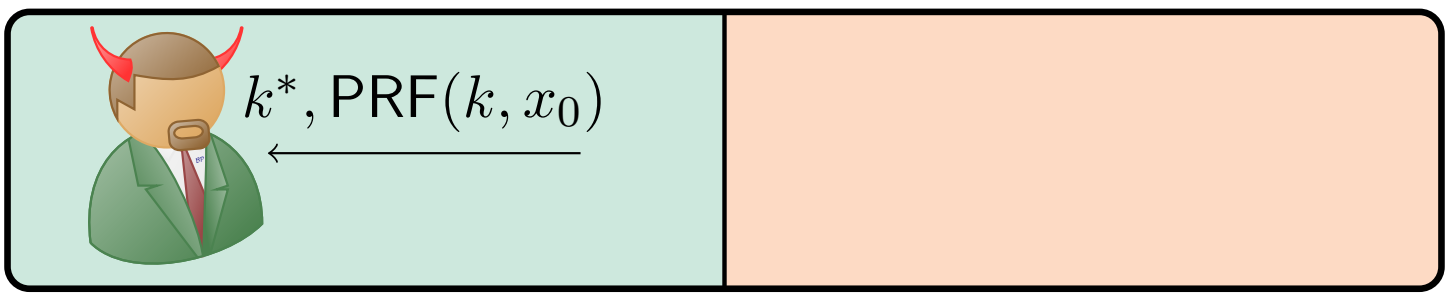

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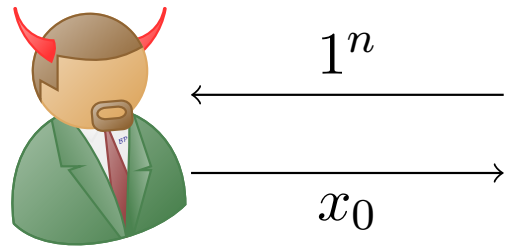

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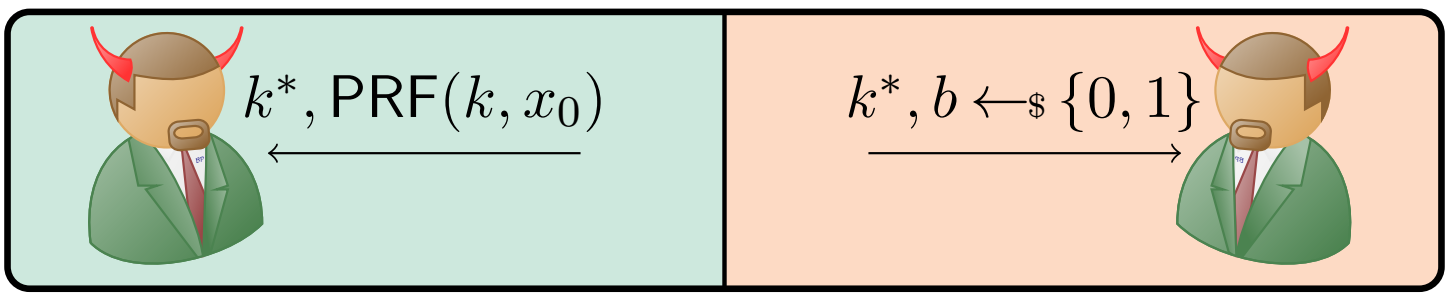

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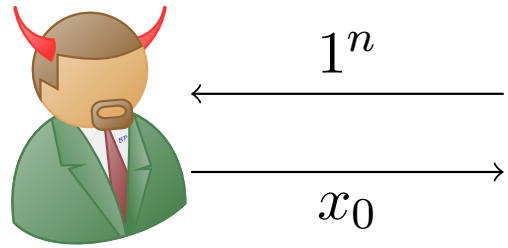

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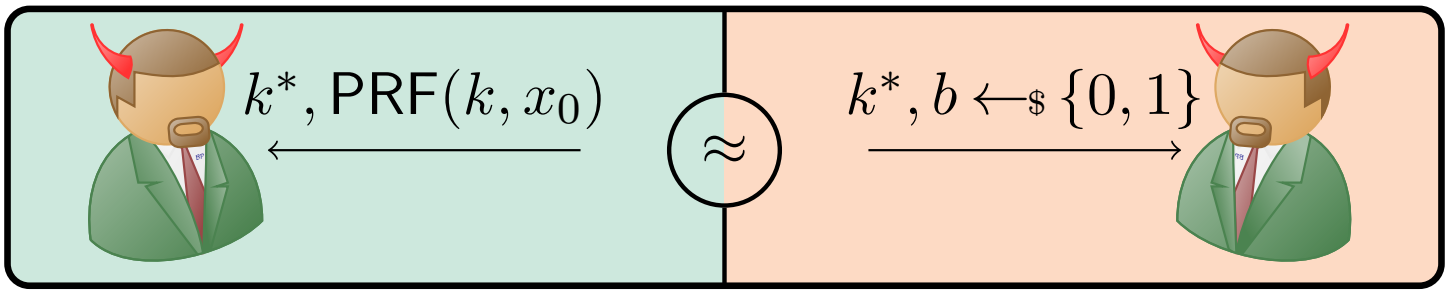

Enforcing Large Statistical Distance

$\mathsf{C}[k^*, x_0, b](x)$

---

if $x = x_0$
  return b
else
  return $\mathsf{PRF}(k^*, x)$

Let C' denote $\mathsf{saiO}(\mathsf{C}[k^*, x_0, b])$.

Enforcing Large Statistical Distance

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$b = \mathsf{PRF}(k, x_0) \oplus 1$

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$\mathsf{C}_{\mathsf{PRF}} \leftarrow \mathsf{saiO}(\mathsf{PRF}(k, \cdot))$
$\Pr[\mathsf{C}_{\mathsf{PRF}}(x_0) = b] \geq 1 - \epsilon$

$b = \mathsf{PRF}(k, x_0)$

$\mathsf{C}[k^*, x_0, b] \equiv \mathsf{PRF}(k, \cdot)$

$\Downarrow$

$\Pr[\mathsf{C}'(x_0) = b] \gtrsim 1 - \epsilon$

Enforcing Large Statistical Distance

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PRF security $\Longleftarrow$

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Enforcing Large Statistical Distance

$\mathsf{C}[k^*, x_0, b](x)$

if $x = x_0$
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Let C' denote $\mathsf{saiO}(\mathsf{C}[k^*, x_0, b])$.

b

$$\begin{aligned} & \Pr[\mathsf{C}_{\mathsf{PRF}}(x_0) = \mathsf{PRF}(k, x_0)] \geq 1 - \epsilon \\ \wedge \quad & \Pr[\mathsf{C}'(x_0) \neq \mathsf{PRF}(k, x_0)] \gtrsim 1 - \epsilon \\ \Longrightarrow \quad & \mathsf{SD}(\mathsf{C}_{\mathsf{PRF}}, C) \gtrsim 1 - 2\epsilon \end{aligned}$$

$, \cdot)$

Pr

$- \epsilon$

Restriction to Unique-SAT

- We restrict our attention to Unique-SAT (USAT)
- USAT is NP hard via a randomized reduction [VV85]
- Combining this with previous results [MX10,BL13] we show that

$$\text{USAT} \in \text{BPP}^{\text{GapSD}} \implies \text{SAT} \in \text{AM} \cap \text{coAM}$$

The Formula-Indexed Circuit

$\mathsf{C}_X[k, s, \Psi](x)$

---

if $\Psi(x \oplus s) = 1$

  return $\mathsf{PRF}(k, x) \oplus 1$

else

  return $\mathsf{PRF}(k, x)$

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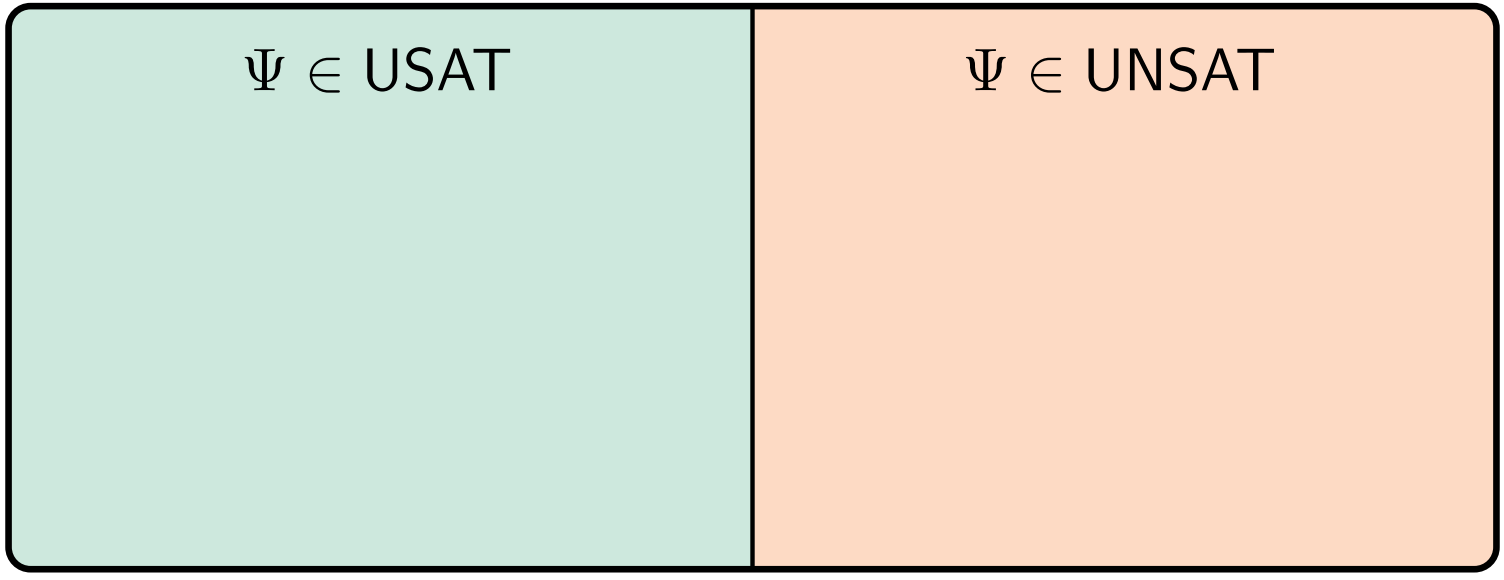

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$\Psi \in \mathsf{USAT}$	$\Psi \in \mathsf{UNSAT}$
	$\mathsf{C}_X[k, s, \Psi] \equiv \mathsf{PRF}(k, \cdot)$

The Formula-Indexed Circuit

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if $\Psi(x \oplus s) = 1$
  return $\mathsf{PRF}(k, x) \oplus 1$
else
  return $\mathsf{PRF}(k, x)$

$\Psi \in \mathsf{USAT}$	$\Psi \in \mathsf{UNSAT}$
$\mathsf{C}_X[k, s, \Psi] \equiv \mathsf{C}[k^*, x_0, b]$ for $x_0 = x_\psi \oplus s$, $b = \mathsf{PRF}(k, x_0) \oplus 1$	$\mathsf{C}_X[k, s, \Psi] \equiv \mathsf{PRF}(k, \cdot)$

Putting it All Together

$X[\Psi](1^n)$

$k \leftarrow\!\!\$\ \{0,1\}^n$

$s \leftarrow\!\!\$\ \{0,1\}^n$

$C := \mathsf{C}_X[k, s, \Psi]$

$C' \leftarrow\!\!\$\ \mathsf{O}(C)$

return (k, s, C')

$Y(1^n)$

$k \leftarrow\!\!\$\ \{0,1\}^n$

$s \leftarrow\!\!\$\ \{0,1\}^n$

$C := \mathsf{PRF}(k, \cdot)$

$C' \leftarrow\!\!\$\ \mathsf{O}(C)$

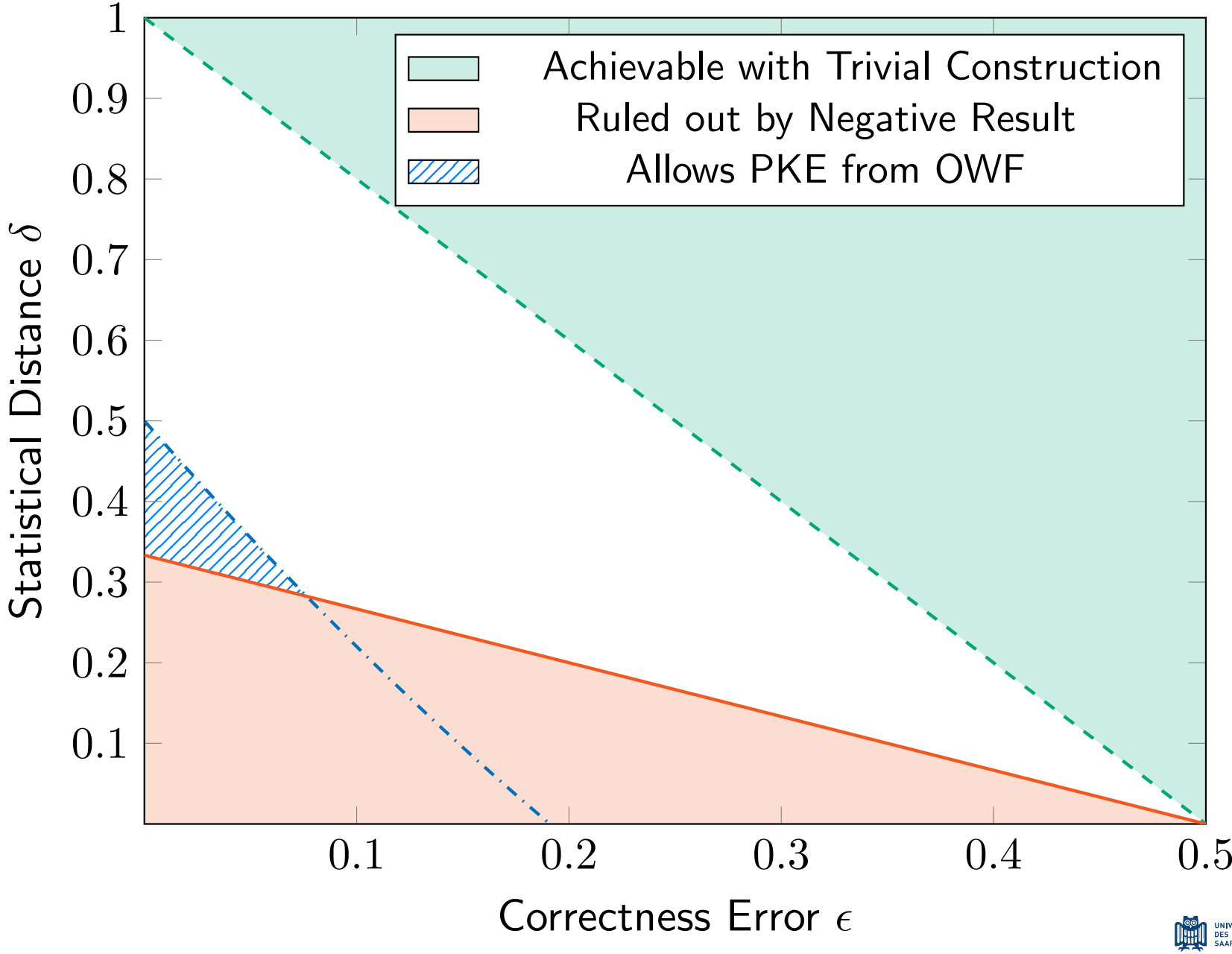
return (k, s, C')

$$\Psi \in \mathsf{UNSAT} \iff \mathsf{SD}(X[\Psi], Y) \leq \mathsf{negl}(n)$$

$$\Psi \in \mathsf{USAT} \iff \mathsf{SD}(X[\Psi], Y) \gtrsim 1 - 2\epsilon$$

- We can therefore decide USAT in $\mathsf{BPP}^{\mathsf{GapSD}}$.
- Thus, if saiO and one-way functions both exist, then $\mathsf{NP} \subseteq \mathsf{AM} \cap \mathsf{coAM}$ and the polynomial hierarchy collapses.

The Landscape of Correlation Obfuscation



Thank You!

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I'm looking for a postdoc position.
If you're interested in hiring me, please send me an email.

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